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Joint Spectrum Allocation and Scheduling in Multi-Radio Multi-Channel Cognitive Radio Wireless Networks

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***Abstract*—Cognitive radio can dynamically adapt to the available spectrum in the wireless network. Scheduling and spectrum allocation are tasks affecting the performance of cognitive radio wireless network. In [1], an iterative approach was proposed to efficiently compute the optimal scheduling for wireless mesh networks with single channel and single radio. The optimal scheduling problem is decomposed to a sequence of small optimization problems and maximum weighted independent set (MWIS) problems, and both of them can be computed quickly even for large networks. For example, the optimal scheduling can be computed for the mesh network with more than 2,000 links in less than one hour. Here, the iterative algorithm is extended to the cognitive radio wireless network with multi-channel and multi-radio. Allowing the schedule problem over multi-channel multi-radio results in higher dimension optimization problem. However, the proposed algorithm can obtain the optimal spectrum allocation and the schedule quickly for moderate size of networks. Numerical experiments show that the optimal throughput is achieved when the number of channels is one or two more than the number of interfaces.**

I. Introduction

The broadcast nature of wireless radio causes interference among the nodes sharing the common communication media. To reduce the interference, fixed spectrum assignment policy regulated by Federal Communications Commission (FCC) assigns a fixed portion of the spectrum to a license holder or a wireless service for exclusive use on a long term basis. However, a large portion of the assigned spectrum is sporadically used or even not utilized at all, which leads to the scarcity of available spectrum. On the other hand, certain portions of the spectrum are heavily used, such as 2.4GHz band and 5.0GHz band.

Dynamic spectrum allocation (DSA) is a good way to share the spectrum among users. The appearance of cognitive radio makes DSA feasible where cognitive radios can be programmed to transmit and receive on a variety of frequencies and use different transmission access technologies. However, how to efficiently share the available spectrum is a challenge in the cognitive radio wireless networks with multi-channel and multi-radio.

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It is well known that the optimal scheduling problem in wireless network is NP-hard even when single channel single radio is considered. The optimization space grows exponentially with the number of links. For example, if single channel is used, then one must maximize over a polytope with 2^L extreme points, where L is the number of links. More specifically, we define an *assignment* to be a specification of which links are transmitting and which links are not transmitting. A *schedule* is the convex sum of assignments. The main challenge to the throughput maximization problem has been that the space of all assignments is too large. In order to solve the scheduling problem, the space of assignments must be reduced. In [1], the authors have developed the iterative techniques to compute the optimal scheduling for the wireless mesh network over single channel even when co-channel interference is present. The idea is as follows. Given an initial set of randomly selected assignments $\mathcal{V}$, solve the throughput optimization problem over the set $\mathcal{V}$ and obtain the Lagrange multiplier for each link. Then, find a better assignment $\mathbf{v}^+$ by solving a maximum weighted independent set (MWIS) problem. If $\mathbf{v}^+$ exists, add it to the set $\mathcal{V}$ and go back to solve the optimization problem again. Otherwise, the optimal scheduling is obtained. The optimality and convergence property are proved in [1].

Allowing the schedule optimization to optimize over multi-channel multi-radio results in a higher dimension problem than when the optimization is over only single channel single radio. However, the main results and the algorithm for optimal scheduling from [1] are still available even if multi-channel and multi-radio are considered. After solving the scheduling problem, the assignment specifies which links are transmitting and their transmission channel. Therefore, DSA problem is also solved as a by-product. In this scheduling problem, the dimension of MWIS problem to finding a new assignment significantly increases with the number of channels. Furthermore, the constraint of multi-radio for each node makes the MWIS problem even harder. The computation complexity of solving MWIS determines the complexity of the scheduling problem over multi-channel multi-radio.

The remainder of this work is organized as follows. Section II briefly introduces some related work. In Section III, notation and problem definition are given. Section IV presents the iterative approach to finding the spectrum allocation and optimal scheduling of the cognitive radio networks when multi-channel multi-radio is supported. The approach focuses on reducing the size of the space of assignments and finding a new assignment. Then, Section V shows the results from numerical experiments. Finally, concluding remarks are given in Section VI.

II. Related Work

Utility optimization of wired networks was pioneered by the seminal works of Kelly [2], Low [3]. The same framework has been extended to ad hoc networks in [4], [5]. The main obstacle facing throughput maximization is that the size over which the optimization is performed is exponential with the number of links [6]–[8] even when single channel single radio is considered. To reduce the optimization space, one approach is to arbitrarily restrict the focus to a subset of all assignments, which is followed in [8]. However, this approach reduces the resulting throughput. Another approach is to develop a heuristic to determine the subset of considered assignments [9], [10]. The performance of these methods typically decreases as the number of links increases [9], [10]. Finally, one can take a brute force approach, and consider the entire set of assignments. This approach is taken in [6], where, due to computational difficulties, the largest network that could be solved has only 15 links. However, the authors in [1] has developed the iterative techniques to compute the optimal scheduling for the wireless mesh network with more than 2,000 links.

When cognitive radio wireless network with multi-channel single-radio is considered, [11] proposed the concept of time-spectrum block and the protocols based on heuristic method to allocate these blocks. [12] also reduced the optimization space by constructing the subset of the good assignments, and obtained the suboptimal from the given assignments. The network topology considered is around 10 links.

Throughput optimization has been studied for wireless mesh network with multi-channel multi-radio. [13]–[15] provided the theoretic results along with the centralized algorithm. Both static and dynamic spectrum allocations are investigated in these works. Due to the large size of optimization space, the approximation approaches were widely used in these works, and the proposed algorithms provided the bounds or the ratio of the capacity instead of the optimal solution. This work proposes an efficient approach to finding the spectrum allocation and scheduling for cognitive radio networks with multi-channel multi-radio by extending the work in [1].

III. Notation and Problem Definition

A router-to-router flow is denoted by ϕ, with Φ denoting the set of all such flows. To improve presentation, it is assumed that all flows use a single path, however, the extension to multipath is straightforward. The data rate of flow ϕ is denoted by f_ϕ, and the path followed by flow ϕ is denoted by $P(\phi)$. The set of all considered paths is $\mathcal{P}$. Using this notation, the total data rate sent over link x is $\sum_{\{\phi | x \in P(\phi)\}} f_\phi$, where $\{\phi | x \in P(\phi)\}$ is the set of flows that cross link x. All links are directed; bidirectional communication between two nodes is represented by two directed links.

Define an *assignment* to be a vector $\mathbf{v} = \left[\begin{array}{ccc} v_1 & \cdots & v_L \end{array}\right]$, where there are L links in the network and where $v_x \in \{0,1\}$ with $v_x = 1$ implying that link x is transmitting during assignment $\mathbf{v}$. The *set of considered assignments* is denoted by $\mathcal{V}$, while the *set of all assignments* is denoted by $\overline{\mathcal{V}}$. In this single channel case where $v_x \in \{0,1\}$, $\overline{\mathcal{V}}$ has 2^L assignments. The size of $\overline{\mathcal{V}}$ is the main challenge facing optimal scheduling. Thus, typically, we only consider a subset of all assignments, i.e., $\mathcal{V} \subsetneqq \overline{\mathcal{V}}$.

Suppose that the available bandwidth B is divided evenly into K orthogonal channels, each having bandwidth $\Delta B = B/K$, sufficiently small to be accurately approximately by a flat fading coefficient. For simplicity, assume all channels have the same fading coefficient. Also, assume that there are L links in the cognitive radio wireless network, and each node x has M_x cognitive radio interfaces. For a given node n_d, all links incident to node n_d are denoted by $E(n_d)$.

When considering multiple channels, the concept of a logical link arises. For example, a physical link may transmit at K orthogonal channels. This can be modeled as K logical links, each with a different channel, but between the same pair of physical nodes. In this case, the logical link x_k would refer to the physical link x transmitting at the kth channel. Then, an assignment specifies which logical links are transmitting. We drop the terms physical and logical unless it is unclear from context.

A schedule is defined as a convex combination of assignments. For example, in the simple case where all links can only transmit at a single channel, then an assignment can be written as $\mathbf{v} \in \{0,1\}^L$, where there are L links in the network. In this case, $v_x = 1$ implies that link x is transmitting during assignment $\mathbf{v}$. A schedule can be represented by a set $\{\alpha_\mathbf{v}\}$ where $\alpha_\mathbf{v} \geq 0$ and $\sum_\mathbf{v} \alpha_\mathbf{v} \leq 1$. Letting $R(\mathbf{v}, x)$ be the data rate over link x during assignment $\mathbf{v}$, the average data rate over link x provided by schedule $\{\alpha_\mathbf{v}\}$ is $\sum_\mathbf{v} \alpha_\mathbf{v} R(\mathbf{v}, x)$. More details on the single channel case can be found in [1]. Since links are permitted to use K channels; hence an assignment can be written as $\mathbf{v} \in \{0,1\}^{L \times K}$, where $v_{x_k} = 1$ implies that the physical link x is transmitting at channel k. Due to the large size of $\{0,1\}^{L \times K}$, we consider subsets $\mathcal{V} \subset \{0,1\}^{L \times K}$, where $\mathcal{V}$ is referred to as the *set of considered assignments*.

The data rate across logical link x_k during assignment $\mathbf{v}$ is denoted by $R(\mathbf{v}, x_k)$. A simple binary approximation is used to define $R(\mathbf{v}, x_k)$. Specifically,

$$R(\mathbf{v}, x_k) = \begin{cases} R_{x_k} \text{ if } v_{y_j} = 0 \text{ for all } y_j \in \chi(x_k), \\ \qquad\qquad y_j \neq x_k, v_{x_k} = 1, \\ 0 \text{ otherwise,} \end{cases} \tag{1}$$

where $\chi(x_k)$ is the set of logical links that are in conflict with x_k, i.e., $y_j \in \chi(x_k)$ if simultaneous transmissions over logical link x_k and logical link y_j are not possible. R_{x_k} is the nominal data rate over logical link x_k. Note that this definition of $R(\mathbf{v}, x_k)$ neglects the possibility that the aggregate interference from transmissions over several links not in $\chi(x_k)$ can result in a transmission failure over logical link x_k. A technique to account for such interference is discussed in Section IV-B. All computations in this work will use this technique, and hence the computed throughputs account for multiple interferers.

The set of conflicting links, $\chi(x_k)$, depends on the communication model. Arguably, the $SINR$ binary communication model is the most relevant and is the model used in this work. Here, the SINR is defined as

$$SINR(x_k, y_j) = \frac{H_{x_k,x_k} p_{x_k}}{H_{y_j,x_k} p_{y_j} + \mathcal{N}},$$

where H_{x_k,x_k} is the channel gain across link x_k, H_{y_j,x_k} is the channel gain from the transmitter of link y_j to the receiver of link x_k, p_{x_k} and p_{y_j} are the transmission powers, and $\mathcal{N}$ is the channel noise. Let us define $\Gamma(x_k)$ be the SINR threshold to achieve the desired data rate R_{x_k}. Under the SINR binary communication model, the set of conflicting links, $\chi(x_k)$ is defined by

$$\begin{aligned} \chi(x_k) \quad = \quad & \{y_j \,| SINR(x_k, y_j) < \Gamma(x_k) \\ & \text{or } SINR(y_j, x_k) < \Gamma(y_j)\}. \end{aligned} \tag{2}$$

Suppose L links share K channels and each node x has M_x cognitive radio interfaces. With this notation, the schedule optimization problem can be described as

$$\max_{\boldsymbol{\alpha}, \mathbf{f}} G(\mathbf{f}) \tag{3a}$$

subject to:

$$\sum_{\{\phi | x \in P(\phi)\}} f_\phi \leq \sum_{\mathbf{v} \in \mathcal{V}} \alpha_{\mathbf{v}} \sum_{k=1}^{K} R(\mathbf{v}, x_k) \text{ for each link } x \tag{3b}$$

$$\sum_{\mathbf{v} \in \mathcal{V}} \alpha_{\mathbf{v}} \leq 1 \tag{3c}$$

$$0 \leq \alpha_{\mathbf{v}} \text{ for each } \mathbf{v} \in \mathcal{V}. \tag{3d}$$

where $\mathbf{f}$ is the vector of flow rates. The function G is referred to as the *throughput metric*. Several different throughput metrics are possible. In some cases, the throughput metric is the network utility $G(\mathbf{f}) = \sum_{\phi \in \Phi} U_\phi(f_\phi)$, where U_ϕ is the utility function for flow ϕ. Popular utility functions include $U_\phi(f) = w_\phi \log(f)$ [2], [16], [17] and $U_\phi(f) = w_\phi f^{1-\gamma} / (1-\gamma)$ [18], where w_ϕ is the administrative weight. Another widely used throughput metric is $G(\mathbf{f}) = \min_{\phi \in \Phi} w_\phi f_\phi$ [8]. If $G(\mathbf{f}) = \min_{\phi \in \Phi} w_\phi f_\phi$, then (3) can be written as a linear programming problem, which is extensively studied in [19]. This work specifically, focuses on the case when $G(\mathbf{f}) = \min_{\phi \in \Phi} w_\phi f_\phi$.

Algorithm 1 Computing an Optimal Schedule

1: Select an initial set of assignments $\mathcal{V}(0)$, set $i = 0$.
2: Solve (3) for $\mathcal{V} = \mathcal{V}(i)$ and compute $\boldsymbol{\mu}(i)$ and $\lambda(i)$, the Lagrange multipliers associated with constraints (3b) and (3c), respectively.
3: Search for an assignment $\mathbf{v}^+$ that solves

$$\mathbf{v}^+ \in \arg\max_{\mathbf{v}} \sum_{x=1}^{L} \mu_x \sum_{k=1}^{K} R(\mathbf{v}, x_k). \tag{4}$$

4: **if** the assignment $\mathbf{v}^+ \in \mathcal{V}(i)$ or $\sum_{x=1}^{L} \mu_x(i) \sum_{k=1}^{K} R(\mathbf{v}^+, x_k) \leq \lambda(i)$ **then**
5:    stop, the optimal schedule has been found.
6: **else**
7:    set $\mathcal{V}(i+1) = \mathcal{V}(i) \cup \mathbf{v}^+$, set $i = i + 1$, and go to Step 2.
8: **end if**

IV. Optimal Scheduling

In theory, (3) is solvable. However, there is a significant computational challenge in that if $\mathcal{V}$ is the set of all assignments, then the vector $\boldsymbol{\alpha}$ has 2^{LK} elements. Thus, the size of the space over which the optimization is performed must be reduced. This idea of considering a reduced space was considered in [8] and [9], however, the space was constructed arbitrarily. In this work the space is constructed so that the throughput found by optimizing over the reduced space is the same throughput found by optimizing over the entire space.

A. Finding New Assignments

In order to find a new assignment, (4) must be solved. It is well known that solving (4) is equivalent to solving the graph theoretic problem known as the maximum weighted independent set (MWIS) problem (e.g., see [1]). In the worst case, the MWIS problem is NP-hard. However, this does not mean that the MWIS problem is computational intractable in all cases. In fact, [20] showed that (4) can be solved efficiently. For example, [20] found that in practical networks, (4) can be solved in about one second for a topology with 2048 links where single channel single radio is considered.

One approach to solving (4) is to use a generic integer linear programming (ILP) solver. The MWIS problem can be formulated as an ILP via

$$\max_{\mathbf{v}} \sum_{x=1}^{L} \mu_x \sum_{k=1}^{K} R_{x_k} v_{x_k} \tag{5a}$$

$$\begin{aligned} \text{subject to: } & v_{x_k} + v_{y_j} \leq 1 \text{ if } y_j \in \chi(x_k) \\ & \sum_{x \in E(y)} \sum_{k=1}^{K} v_{x_k} \leq M_y \text{ for each node } y \quad (5b) \\ & v_{x_k} \in \{0,1\}. \end{aligned}$$

where $x \in E(y)$ denotes the links incident to node y and (5b) restricts that the active links incident to node y must be no more than the number of cognitive radios in node y. The time required to solve this problem can be significantly decreased if a greedy clique decomposition of the conflict graph is performed. Specifically, a set of cliques $\{Q_i,\ i = 1, 2, ...\overline{Q}\}$ are found such that if $y_j \in \chi(x_k)$, then there is a clique Q_i such that $x_k \in Q_i$ and $y_j \in Q_i$. Then, (5a) becomes

$$\max_{\mathbf{v}} \sum_{x=1}^{L} \mu_x \sum_{k=1}^{K} R_{x_k} v_{x_k} \tag{6}$$

$$\begin{aligned} \text{subject to: } & \sum_{x_k \in Q_i} v_{x_k} \leq 1 \text{ for } i = 1, 2, ..., \overline{Q} \\ & \sum_{x \in E(y)} \sum_{k=1}^{K} v_{x_k} \leq M_y \text{ for each node } y \\ & v_{x_k} \in \{0,1\}. \end{aligned}$$

B. Removing Multi-Conflict

Binary communication models have the drawback that they do not directly account for the impact of the aggregate of interference from several simultaneously transmitting links. For example, suppose that links $y_j \notin \chi(x_k)$, $z_l \notin \chi(x_k)$, $y_j \notin \chi(z_l)$, thus, according to the binary conflict model, any pair of these three links can simultaneously transmit. However, it is possible that the aggregate of the interference of link y_j and link z_l transmitting is strong enough that communication across link x_k is not possible. However, binary conflict models do not account for this type of interference. Consequently, a schedule constructed based a binary conflict model may perform poorly when applied in the physical model.

One way to account for these multi-conflict is as follows. Let $\mathbf{v}^+$ be a new assignment found by solving (6). Check whether this assignment has any multi-conflict, by determining whether there is a x_k with $v^+_{x_k} = 1$ and

$$\frac{H_{x_k,x_k} p_{x_k}}{\sum_{y=1,y\neq x}^{L} \sum_{j=1}^{K} H_{y_j,x_k} p_{y_j} v_{y_j} + \mathcal{N}} < \Gamma(x_k).$$

If such a x_k exists, then add the found multi-conflict into the ILP (6) as follows. Let $\mathcal{C} = \{x_k | v^+_{x_k} = 1\}$, that is, $\mathcal{C}$ is the set of links that make up the multi-conflict. Intuitively, C should be the smallest set of links that form the multi-conflict at link x_k. This multi-conflict can be removed from consideration by using the following ILP problem to search for new assignments,

$$\max_{\mathbf{v}} \sum_{x=1}^{L} \mu_x \sum_{k=1}^{K} R_{x_k} v_{x_k} \tag{7a}$$

$$\begin{aligned} \text{subject to: } & \sum_{x_k \in Q_i} v_{x_k} \leq 1 \text{ for } i = 1, 2, ..., \overline{Q} \\ & \sum_{x \in E(y)} \sum_{k=1}^{K} v_{x_k} \leq M_y \text{ for each node } y \\ & \sum_{x_k \in \mathcal{C}} v_{x_k} \leq |\mathcal{C}| - 1 \quad (7b) \\ & v_{x_k} \in \{0,1\}, \end{aligned}$$

where $|\mathcal{C}|$ is the number of elements in $\mathcal{C}$. Again, it is determined whether the assignment found by solving (7) has multi-conflict and if so, another constraint such as (7b) is added to the ILP. This process continues until an assignment is found that does not have a multi-conflict.

V. Numerical Experiments

A. Computational Experimental Set-up

In order to determine the performance of the various techniques described above, this investigation employed the realistic urban mesh networks constructed by the UDelModels [21]. Along with a realistic propagation simulator, the UDel Models include a map builder, a realistic mobility simulator, and a large collection of data and trace files. The propagation simulator is based on ray-tracing and accounts for reflections off of the ground and off of buildings, transmission through building walls, and diffraction around and over buildings [22]. It also accounts for the impact that different materials have on reflections off of walls and transmission through walls. Data sets for several urban areas are available online.

For this investigation, the topologies are generated as follows. Each topology was based on a different 6×6 city block region that was randomly located within a 2 km^2 region of downtown Chicago. Various node densities were investigated. Specifically, the number of gateways[1] was 1, 3, and 6, and the number of wireless routers was 18, 36, 54, 72, and 90. The wireless routers and gateways were uniformly distributed throughout the 6×6 city block region.

In these experiments, all traffic flowed from the gateways to destinations (i.e., downstream traffic), where each mesh router in the topology was a destination of a flow. The routing was a single path, least hop routing, where each link had a receiver signal strength of at least -75 dBm when the transmission power was 15 dBm.

[1]Gateways have both wired and wireless interfaces, whereas wireless routers only have wireless interfaces.

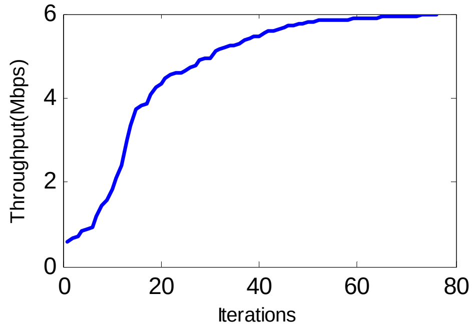

Fig. 1. Variation in the computed throughput as assignments are added where the throughput is $\min_{\phi\in\Phi} f_\phi$. The plot is for a topology with 3 GW and 90 wireless routers where the number of channels is 7 and the number of radios at each node is 4.

802.11a data rates were used. Furthermore, it was assume that the relationship between SNR and bit-rate is the same as the relationship between SINR and bit-rate. The throughput metric used is $G(\mathbf{f}) = \min_{\phi\in\Phi} f_\phi$. The computations below were performed on a 2.4MHz AMD FX-53 processor with 8GB RAM. Programs were written in Matlab and used CPLEX v10 for solving LP and ILP problems.

B. Number of Iterations until Algorithm 1 Stops

Figure 1 shows how, in the topology with 3 GW and 90 wireless routers where the number of channels is 7 and the number of radios at each node is 4, the throughput increases as the more assignments are added. The point of maximum throughput occurs when the stopping condition specified in Algorithm 1 is met. Thus, in this case, Algorithm 1 stopped after 76 iterations when the throughput metric was $G(\mathbf{f}) = \min_{\phi\in\Phi} f_\phi$.

Note that only one assignment is added at each iteration. Thus, the maximum number of elements in $\mathcal{V}$ is the number of assignments in the initial set of assignments plus the number of iterations required by Algorithm 1. Hence, we have achieved the goal of determining the solution to (3) for $\mathcal{V} = \bar{\mathcal{V}}$ by computing the solution to (3) for a small set $\mathcal{V}$. The complexity of solving linear and nonlinear optimization problems is well known, and is not investigated here.

C. Multi-Channel and Multiple Cognitive Radios

The proposed framework easily covers the multi-channel multi-radio scenario. Assume that all nodes have the same number of interfaces. Figure 2 shows the relationships among the throughput, the number of channels and the number of interfaces. The y-axis is the throughput ratio over the optimal throughput with single channel and single radio. These plots are for a wireless network with 3 GWs and 36 wireless routers. A few comments are in order.

- If there are n channels available and n radios at each node, the throughput increases with a factor of n because all channel are same as each other.

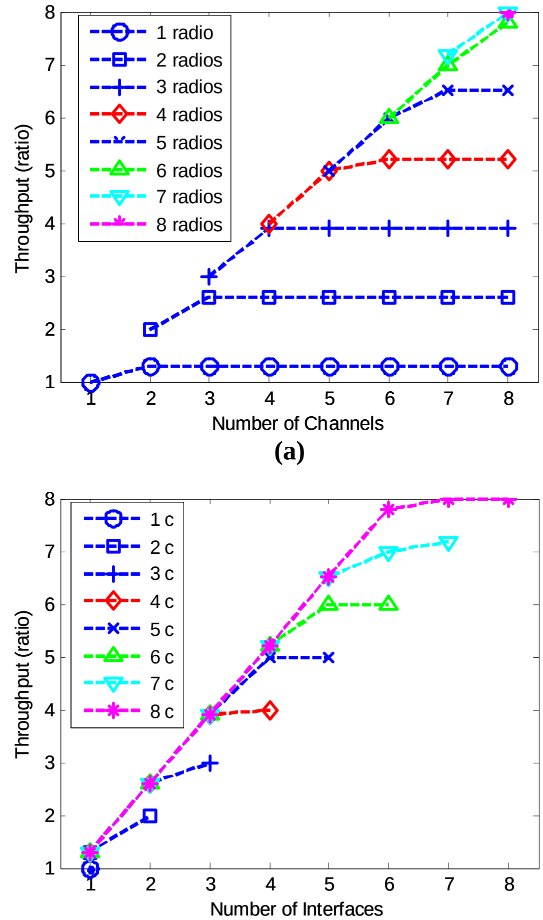

Fig. 2. (a). Throughput versus the number of channels with different number of interfaces. (b). Throughput versus the number of interfaces with different number of channels. These plots are for a wireless network with 3 GWs and 36 wireless routers.

- The optimal throughput is achieved when the number of channels is one or two more than the number radios. The throughput is not improved any more even if the number of channel increases further.
- The throughput of two channels and single radio is better than that of single channel and single radio. Similar results are correct for $n+1$ channels and n radios case.

The results are also true for other generated topologies.

VI. Conclusion

In this paper, we presented the problem of joint spectrum allocation and scheduling in multi-radio multi-channel cognitive radio wireless networks. The iterative algorithm proposed in [1] was extended to solve the optimization problem with higher dimension. Specifically, with the introduction of logical links, the dimension of the MWIS problem is increased by K times where K is the number of channels, while the optimization problem (3) over a small set of assignments still has the same dimension. With this iterative algorithm, we can efficiently compute the joint spectrum allocation and scheduling for the wireless networks with a few hundred links. In addition, numerical experiments show that the optimal throughput is achieved when the number of channels is one or two more than the number of interfaces.

DISCLAIMER

This work was performed while the first author held a National Research Council Research Associateship Award at the Air Force Research Laboratory.

The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Air Force Research Laboratory or the U. S. Government.

REFERENCES

[1] P. Wang and S. Bohacek, "Practical computation of optimal schedules in multihop wireless networks," submitted 2007, available at http://udelmodels.eecis.udel.edu.

[2] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: Shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, pp. 237–252, Nov 1998.

[3] S. H. Low and D. E. Lapsley, "Optimization flow control.i: Basic algorithm and convergence," *IEEE/ACM Transactions on Networking*, vol. 7, no. 6, pp. 861–874, 1999.

[4] M. Chiang, "To layer or not to layer: Balancing transport and physical layers in wireless multihop networks," in *IEEE Infocom*, 2004.

[5] Y. Yi and S. Shakkottai, "Hop-by-hop congestion control over a wireless multi-hop network," in *IEEE Infocom*, 2004.

[6] R. Cruz and A. Santhanam, "Optimal routing, link scheduling and power control in multi-hop wireless networks," in *IEEE Infocom*, March 2003, pp. 702–711.

[7] L. Chen, S. H. Low, and J. C. Doyle, "Joint congestion control and media access control design for wireless ad hoc networks," in *Proceedings of IEEE INFOCOM*, March 2005, pp. 2212–2222.

[8] K. Jain, J. Padhye, V. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," in *Proceedings of ACM MobiCom*, San Diego, CA, September 2003, pp. 66–80.

[9] T. ElBatt and A. Ephremides, "Joint scheduling and power control for wireless ad-hoc networks," in *Proceedings of IEEE INFOCOM*, New York, NY, June 2002, pp. 976–985.

[10] G. Brar, D. M. Blough, and P. Santi, "Computationally efficient scheduling with the physical interference model for throughtput improvment in wireless mesh networks," in *Mobicom*, 2006, pp. 2–13.

[11] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, "Allocating dynamic time-spectrum blocks for cognitive radio networks," in *MobiHoc07*, 2007, pp. 130–139.

[12] J. Tang, S. Misra, and G. Xue, "Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks," *Computer Networks*, vol. 52, pp. 2148–2158, 2008.

[13] M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *MobiCom*. ACM, 2005.

[14] P. Kyasanur and N. H. Vaidya, "Capacity of multi-channel wireless networks: Impact of number of channels and interfaces," in *ACM Mobicom*, 2005.

[15] M. Alicherry, R. Bhatia, and L. Li, "Joint channel assignment and routing for throughout optimization in multi-radio wireless mesh networks," in *MobiCom*. ACM, 2005.

[16] F. P. Kelly, "Charging and rate control for elastic traffic," *European Transactions on Telecommunications*, vol. 8, pp. 33–37, January 1997.

[17] S. H. Low, "A duality model of TCP and queue management algorithms," *IEEE/ACM Transactions on Networking*, vol. 11, pp. 525–536, August 2003.

[18] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 556–567, 2000.

[19] J. Nocedal and S. Wright, *Numerical Optimization*. Springer, 2000.

[20] P. Wang and S. Bohacek, "On the practical complexity of solving the maximum weighted independent set problem for optimal scheduling in wireless networks," in *WICON*, 2008.

[21] S. Bohacek, V. Sridhara, and J. Kim, "UDel Models," available at: http://udelmodels.eecis.udel.edu/.

[22] V. Sridhara and S. Bohacek, "Realistic propagation simulation of urban mesh networks," *The International Journal of Computer and Telecommunications Networking Computer Networks and ISDN Systems (COMNET)*, vol. 51, no. 12, pp. 3392–3412, 2007.